A study on behavior of the Latin American Tower subjected to wind loads

Iván Huergo Ríos[1*], Hugo Hernández Barrios[2]
[1*] *PhD student of the Faculty of Engineering, UNAM, Mexico (ing.huergo@hotmail.com)*
[2] *Civil Engineering Postgraduate Building, Michoacan University of Saint Nicholas of Hidalgo, Mexico (hugohernandezbarrios@yahoo.com.mx)*

***Abstract*– The Latin American Tower was built in the center of Mexico City in 1956, becoming the tallest building in Latin America at the time. The wind loads considered in the design of the skyscraper were based on a wind pressure of 686.54 *Pa*. At that time there were not enough procedures to simulate wind speed time series, that is why the structural engineers used wind static loads on the design of The Latin American Tower. Currently due to climate change and a better understanding of wind loads, the Mexican Technical Norms relating to wind resistant design of structures suggest reference wind velocities. This paper shows the along-wind dynamic response of the Latin American Tower through a simplified model of masses and springs subjected to wind speed time series. The results shows an increase in the displacements, shear forces and base overturning moment of the Latin American Tower with respect to those initially obtained during its static wind design made in the 1950s. The along-wind dynamic response of the skyscraper in the dynamic analysis almost exceeds the serviceability limit states under wind loads recommended by current standards.**

1. INTRODUCTION

The Latin American Tower was the first skyscraper built in an area of high seismic risk and became one of the safest buildings in the world.

The building was built in Mexico City in 1956 and it has a total height of 181.33 *m* with 44 floors, an antenna and a lightning rod placed on its top. The structure was made of steel with concrete slabs and its cross section is square varying with the height.

This skyscraper was designed under the action of gravity and lateral loads (earthquake and wind). The seismic design of the building was based on a dynamic spectral analysis, while the wind design was based on a static analysis.

Research on wind turbulence began around the middle of the 20th century, this is the reason why most of the buildings built at that time were designed only by static wind loads, just like the Latin American Tower.

The current wind loads have increased due to climate changes. Therefore, the static wind analysis made in the 1950s for the Latin American Tower is compared in this paper with a time domain dynamic analysis in the along-wind direction.

2. STATIC WIND ANALYSIS MADE IN THE 1950s

The Latin American Tower was designed in the 1950s through wind static loads. A general view of the skyscraper is shown in Figure 1.

Fig. 1: General view of the Latin American Tower

The experiments carried out at that time to determine the pressure of the wind on areas of 2.32 to 9.29 m^2 resulted in the experimental formula [1]:

$$U = \sqrt{\frac{P}{0.9535}} \tag{1}$$

Where U is the wind velocity in m/s and P is the wind pressure in Pa. Based on equation (1), the wind was classified as shown in table 1.

Table. 1: Classification of the wind in the 1950s [1]

Description of the wind	U (m/s)	P (Pa)
Calm wind	0.28	0.074
Breeze	1.67	2.65
Fresh wind	11.11	117.72
Strong wind	19.44	360.51
Very strong wind	22.22	470.86
Dangerous wind	27.78	735.73
Hurricane	44.44	1883.45

The static wind design of the Latin American Tower was made with a pressure of 686.54 Pa corresponding to a wind velocity, U, of 26.83 m/s. This wind pressure was applied from the floor 44 to 13, while it changed from 686.54 to 0 P_a on the lower floors.

The height of the Latin American Tower is 139 m from the ground to the 44th floor, while from the ground to the antenna is 165.97 m, and from the ground to the lightning rod is 181.33 m. The static wind forces were only considered from the ground to the 44th floor because the antenna and the lightning rod do not have exposed areas to the wind. The skyscraper was modeled as a spring-mass system where the lumped mass and lateral spring stiffness of each floor were calculated by Dr. Nathan M. Newmark at the request of Dr. Leonardo Zeevaert [1]. The static displacements in the along-wind direction were obtained by the equation:

$$\{y\} = [K]^{-1}\{F\} \tag{2}$$

Where:

$$F = \frac{1}{2} p_a \, A \, C_D \, U^2 \tag{3}$$

$$p_a = p_o \frac{0.392\Lambda}{273+\tau_e} \tag{4}$$

$$p_o = 1.225 \; kg/m^3 \tag{5}$$

In equation (2), $\{y\}$ is a vector with the static displacements in the along-wind direction, $[K]$ is the stiffness matrix of the building and $\{F\}$ is a vector with the static wind forces in the along-wind direction. In equation (3) to (5), A is the wind exposed area of each floor, whose values were obtained by a 3d model from Google- SketchUp; C_D is the drag coefficient which is approximately equal to 1.2 when the cross section of the building is square like the Latin American Tower; p_a is the air density in kg/m^3 of the place of the structure; p_o is the air density at sea level and at a temperature of 15°C; Λ is the barometric pressure in mm of mercury of the place of the structure; and τ_e is the global annual temperature in °C of the place of the structure. For Mexico City, Λ is equal to 586 mm of mercury, and τ_e is equal to 16°C.

The displacements induced by along-wind static forces are shown in Figure 2.

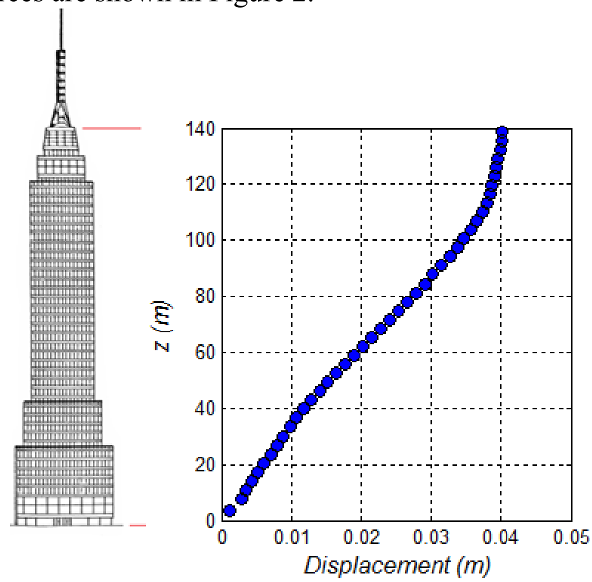

Fig. 2: Wind static displacements of the Latin American Tower according to the original design made in the 1950s

The shear force at the base, V_B, and the base overturning moment of the building, M_B, are shown in table 2 for the static wind forces in the along-wind direction.

Table. 2: Static stresses at the base of the Latin American Tower according to the original design made in the 1950s

V_B	938.18 kN
M_B	67,200.62 $kN \cdot m$

3. ALONG-WIND RESPONSE ANALYSIS IN TIME DOMAIN

The Complementary Technical Norms for wind design of Mexico City [2] were elaborated in the year 2004. These norms contemplate changes in the reference wind velocity with respect to the 1950s and propose the power law to calculate the mean wind velocity at different heights. However, the power law and the logarithmic law are not valid at very high altitudes above ground. Therefore, the profile of mean wind velocity was calculated through the corrected logarithmic profile proposed by Harris and Deaves [3]:

$$\bar{U}(z)=\frac{u^{*}}{k}\left[\ln\left(\frac{z}{z_o}\right)+5.75\,a-1.88\,a^{2}-1.33\,a^{3}+0.25\,a^{4}\right] \tag{6}$$

Where:

$$k\approx 0.4 \tag{7}$$

$$u^{*}\approx\frac{k\,\bar{U}(z_r)_{D,10\min}}{\ln(z_r/z_o)} \tag{8}$$

$$z_r=10\,m \tag{9}$$

$$\bar{U}(z_r)_{D,10\min}=F_T\,F_R\,\bar{U}(z_r)_{R,10\min} \tag{10}$$

$$\bar{U}(z_r)_{R,10\min}=\frac{\bar{U}(z_r)_{R,3\,s}}{1.42} \tag{11}$$

$$F_R=k_t\,\ln\left(\frac{z_r}{z_o}\right) \tag{12}$$

$$a=\frac{z}{z_g} \tag{13}$$

$$z_g=\frac{u^{*}}{6\,f_c} \tag{14}$$

$$f_c=2\,\Omega\,sen(\lambda) \tag{15}$$

In equations (6) to (15), $\bar{U}(z)$ is the mean wind velocity, u^{*} is the friction velocity, k is the Von Kármán constant, z_o is the roughness length, z is the height above ground, z_r is the reference height, $\bar{U}(z_r)_{R,3\,s}$ is the reference wind velocity averaged every 3 *s* on a terrain category 2, $\bar{U}(z_r)_{R,10\min}$ is the reference wind velocity averaged every 10 *min* on a terrain category 2, $\bar{U}(z_r)_{D,10\min}$ is the reference wind velocity averaged every 10 *min* on the terrain category where the structure is located, F_T is the topography factor, F_R is the roughness factor, k_t is a roughness parameter, z_g is the gradient height, f_c is the Coriolis parameter in s^{-1}, Ω is the angular velocity of the Earth $(7.27\cdot 10^{-5}\,rad/s)$ and λ is the latitude in ° of the place where the structure is located. The values of some of these parameters are proposed by the Eurocode 1 [4], which are shown in table 3.

Table. 3: Terrain roughness parameters

Terrain	Category	z_o (m)	$z_{\min}$ (m)	k_t	F_R
Desert	1	0.01	2	0.17	1.17
Open spaces	2	0.05	4	0.19	1
Suburban	3	0.3	8	0.22	0.77
Urban	4	1	16	0.24	0.55

Where $z_{\min}$ is the height below which the mean wind velocity can be taken as constant. The Latin American Tower is located in the center of Mexico City $(\lambda=19.42°)$ on a terrain category 4. There are not major topography changes, therefore the value of F_T can be taken as 1. The value of $\bar{U}(z_r)_{R,3\,s}$ for the Latin American Tower was taken from The Complementary Technical Norms for wind design of Mexico City [2] for a period of return of 50 years, whose value is equal to 36 *m/s*.

A dynamic analysis in time domain requires of the simulation of wind time series, $u(z,t)$, which represent the turbulence of along-wind velocity with a standard deviation of 0 *m/s*. The total duration of the simulated wind time series was 10 *min* and the total along-wind velocity is represented as $U(z,t)$, as shown in Figure 3.

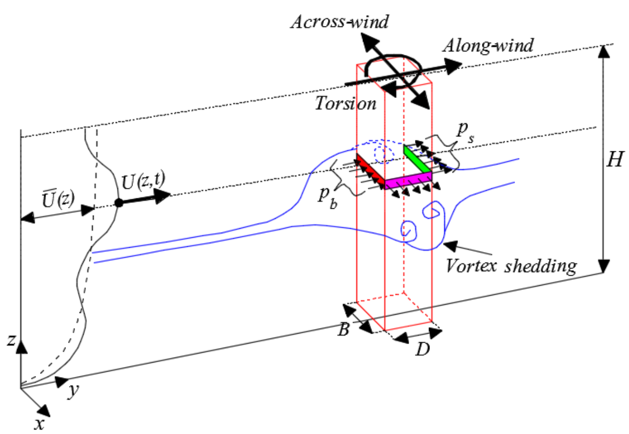

Fig. 3: Dynamic effects of wind on a skyscraper

Where:

$$U(z,t)=\bar{U}(z)+u(z,t) \tag{16}$$

Turbulent wind can be represented as the sum of a large number of waves, which have different amplitude and frequency, $f\ (Hz)$. Therefore, the power spectrum of the fluctuation of the wind speed, $S_u(z, f)$, describes the energy of each wave. In this paper, it was used the power spectrum proposed by Solari [5]:

$$S_u(z, f) = \frac{6.868\ \sigma_u^{\,2}(z)\, L_u(z)/\bar{U}(z)}{\left[1+10.302\left(\dfrac{f\, L_u(z)}{\bar{U}(z)}\right)\right]^{5/3}} \quad (17)$$

Where:

$$\sigma_u(z) = \sqrt{\beta}\ u^* \quad (18)$$

$$\beta = 4.5 - 0.856\ \log(z_o) \quad (19)$$

$$L_u(z) = 300\left(\frac{z}{200}\right)^{v} \quad (20)$$

$$v = 0.67 + 0.05\ln(z_o) \quad (21)$$

In equation (18) and (19), $\sigma_u(z)$ is the expected standard deviation of the turbulent wind and β is a constant which depends on the roughness length [6]. On the other hand, $L_u(z)$ is the expected integral length scale proposed by Solari and Piccardo [7] in equation (20). The turbulence intensity of the total along-wind velocity, $I_u(z)$, was obtained by equation:

$$I_u(z) = \frac{\sigma_u(z)}{\bar{U}(z)} \quad (22)$$

The bandwidth of $S_u(z, f)$ was delimited by $f_1 = 0.001\,Hz$ and $f_u = 10\,Hz$, which are values that include the characteristic frequencies of the wind gusts and the vibration frequencies of any skyscraper. According to the Nyquist theorem, the time interval for the simulation of wind time series is:

$$\Delta t = \frac{1}{2\, f_u} \quad (23)$$

And the frequency interval is:

$$\Delta f = \frac{f_u - f_1}{N-1} = \frac{f_u - f_1}{\left[(T_s/\Delta t)+1\right]-1} \quad (24)$$

Where N is the number of values of the discrete signal and T_s is the total duration of the discrete signal $(T_s = 600\,s)$. The theoretical power spectrums of equation (17) were transformed in function of circular frequencies, w, by dividing them between 2π.

For a set of m stationary random signals $u_j(z,t)$, with zero mean and $j = 1,2,3,...,m$, the cross-spectral density matrix was obtained as a function of the circular frequency, w, by the equation:

$$S^0(w) = \begin{bmatrix} S^0{}_{11}(w) & S^0{}_{12}(w) & \cdots & S^0{}_{1m}(w) \\ S^0{}_{21}(w) & S^0{}_{22}(w) & \cdots & S^0{}_{2m}(w) \\ \vdots & \vdots & \ddots & \vdots \\ S^0{}_{m1}(w) & S^0{}_{m2}(w) & \cdots & S^0{}_{mm}(w) \end{bmatrix},\quad 0 \le w \le \infty \quad (25)$$

Where:

$$S^0{}_{jk}(w) = \sqrt{S^0{}_{jj}(w)\, S^0{}_{kk}(w)}\ \sqrt{Coh_{jk}(w)} \quad (26)$$

$$\sqrt{Coh_{jk}(w)} = \exp\left(-\frac{2\, w\, C_z\, r_z}{\bar{U}(z_j)+\bar{U}(z_k)}\right) \quad (27)$$

$$r_z = \left|z_k - z_j\right| \quad (28)$$

In the equations (26) to (28), *j* and *k* represent two points in the space separated by a vertical distance r_z, $S^0{}_{jj}(w)$ and $S^0{}_{kk}(w)$ represent respectively the functions $S_u(z, w)$ in the vertical points j and *k*.

On the other hand, $\sqrt{Coh_{jk}(w)}$ represent the root of the coherence function proposed by Davenport [8], where $\bar{U}(z_j)$ is the mean wind velocity at point *j*, $\bar{U}(z_k)$ is the mean wind velocity at point *k*, and C_z is the vertical decay constant. The typical value of C_z used in this paper is the one proposed by Dyrbye and Hansen [9], which is equal to 10.

The Cholesky decomposition was applied to the cross-spectral density matrix by the equation:

$$S^0(w) = H(w)\, H^T(w) \quad (29)$$

Where:

$$H(w) = \begin{bmatrix} H_{11}(w) & 0 & \cdots & 0 & \cdots & 0 \\ H_{21}(w) & H_{22}(w) & & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots & & \\ H_{j1}(w) & H_{j2}(w) & \cdots & H_{jj}(w) & & 0 \\ \vdots & \vdots & & \vdots & \ddots & \\ H_{m1}(w) & H_{m2}(w) & \cdots & H_{mj}(w) & \cdots & H_{mm}(w) \end{bmatrix} \quad (30)$$

The synthetic wind time series, $u(z,t)$, can be obtained by the spectral representation method proposed by Shinozuka [10] through the use of the lower Cholesky matrix of the equation (30). Therefore, the simulations were obtained by equation:

$$u_j(z,t)=\sum_{k=1}^{m}\sum_{n=1}^{N}\left|H_{jk}(w_n)\right|\sqrt{2\Delta w}\cos\left[w_n t+\varphi_{kn}\right] \tag{31}$$

Where w_n is the n^{th} circular frequency in the bandwidth of $S_u(z,w)$, and φ_{kn} is a random phase uniformly distributed on the interval $[0,2\pi]$.

The simulations $U(z,t)$ were obtained through the equation (16) once the simulations $u(z,t)$ were obtained on each floor of the building by the equation (31). The simulated values of $\sigma_u(z)$, $\bar{U}(z)$, and $L_u(z)$ were obtained for each signal by the equations:

$$\sigma_u(z)=\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}u_i(z)^2} \tag{32}$$

$$\bar{U}(z)=\frac{1}{N}\sum_{i=1}^{N}U_i(z) \tag{33}$$

$$L_u(z)=T_u(z)\,\bar{U}(z)=\left[\int_0^{\tau_o}R(z,\tau)\,d\tau\right]\bar{U}(z) \tag{34}$$

Where:

$$R(z,\tau)=\frac{1}{\sigma_u{}^2(z)(N-r)}\sum_{i=1}^{N-r}u_i(z)\,u_{i+r}(z) \tag{35}$$

$$\tau=r\,\Delta t \tag{36}$$

In equations (34) to (36), $T_u(z)$ is the time scale, $R(z,\tau)$ is the autocorrelation function of the simulated signal, τ_o is the first root of the autocorrelation function, τ is the lag time and r is the lag number.

In order to solve the imperfect averaging effect associated with the discretized nodes of the building, the aerodynamic admittance function was also incorporated. Castro [11] made a slightly modification of the aerodynamic admittance function proposed by Vickery [12], which is shown in the equation:

$$\left|\chi(z,f)\right|^2=\left[1+\left(\frac{2f\sqrt{A}}{\bar{U}(z)}\right)^{4/3}\right]^{-7/6} \tag{37}$$

The simulations of wind velocity were taken to the frequency domain through the fast Fourier transform in order to be multiplied by the filter of the equation (37). Once the simulations were filtered, they were returned to the time domain by the inverse fast Fourier transform.

Then, the simulations of wind velocity were transformed into nodal forces applied to each floor of the building by equation:

$$F(z,t)=\frac{1}{2}p_a A C_D\left[\bar{U}(z)+u(z,t)\right]^2 \tag{38}$$

A parametric analysis of the simulations was carried out, which can be observed in figure 4.

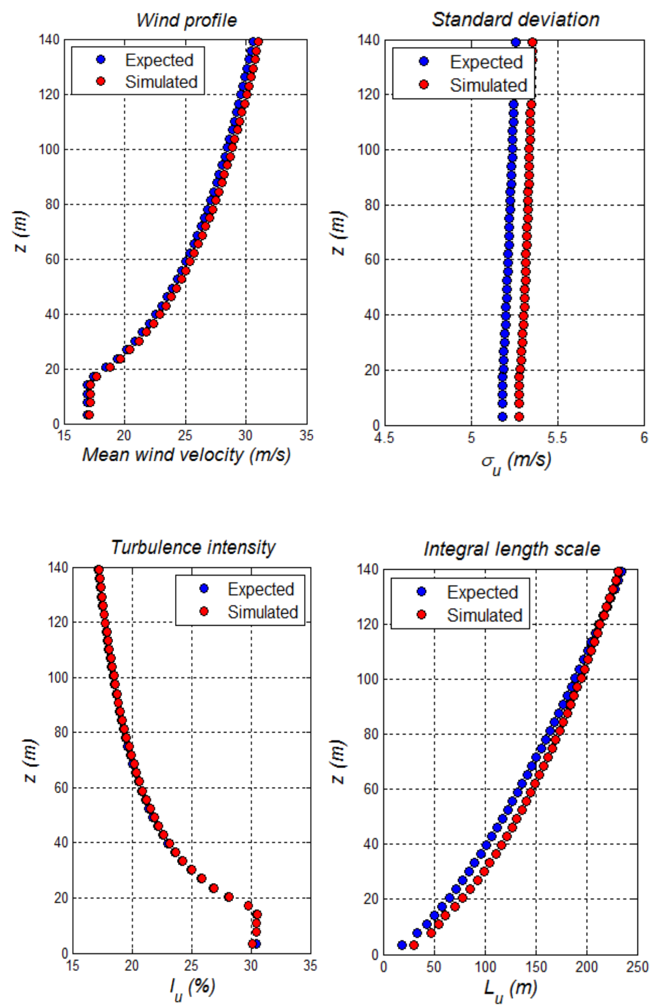

Fig. 4: Parametric analysis of the wind time series for the Latin American Tower

The energy content of some simulations of wind velocity is shown in figure 5.

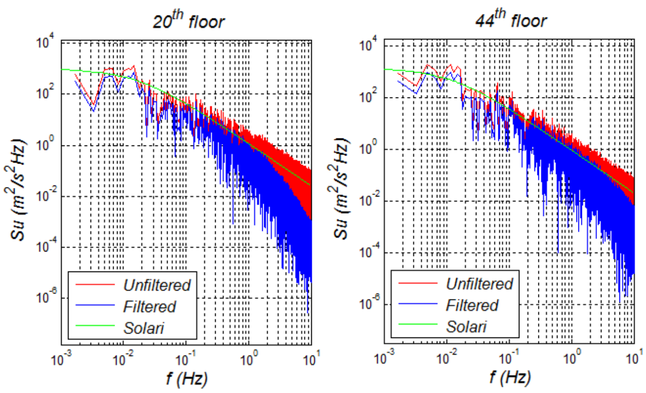

Fig. 5: Power spectrums of wind velocity for the Latin American Tower

Some simulations of the wind velocity for the Latin American Tower are shown in figure 6:

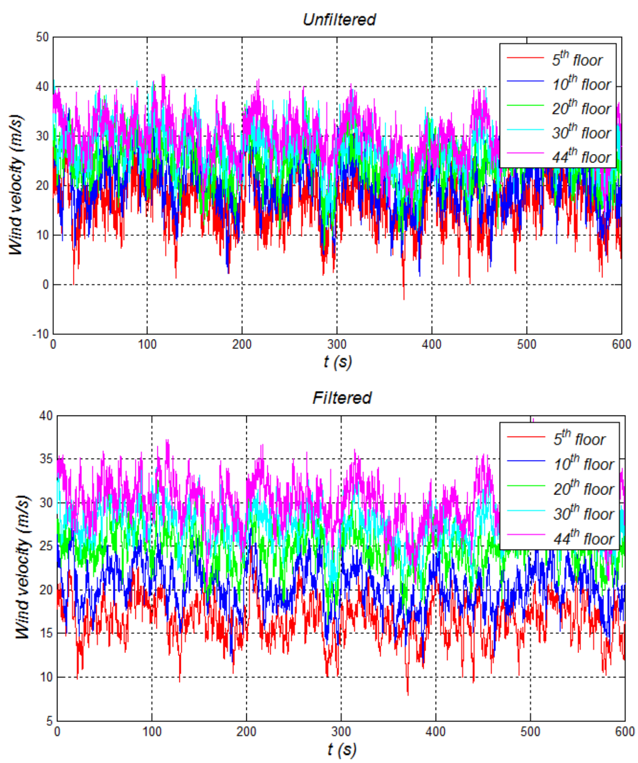

Fig. 6: Simulations of $U(z,t)$ for the Latin American Tower

The dynamic response of the skyscraper is described by the system of equations:

$$[M]\{\ddot{y}(t)\}+[C]\{\dot{y}(t)\}+[K]\{y(t)\}=\{F(t)\} \quad (39)$$

Where the mass matrix, $[M]$, and the stiffness matrix , $[K]$, were obtained from the spring-mass system developed by Newmark for the Latin American Tower [1]. The typical value of structural damping ratio for steel buildings is 1% in wind engineering. A Rayleigh damping matrix, $[C]$, was used in equation (39) with a damping ratio, ς, of 1% for the first and second modes of vibration. The equation (39) was solved by the state-space approach for every i^{th} instant of time by the equation:

$$q_{i+1}=e^{A\Delta t}q_i+e^{A\Delta t}\Delta t\,F_{e,i} \quad (40)$$

Where:

$$A=\begin{bmatrix}[0] & [I]\\ [-M^{-1}K] & [-M^{-1}C]\end{bmatrix} \quad (41)$$

$$F_e(t)=\begin{Bmatrix}[0]\\ [M^{-1}F(t)]\end{Bmatrix} \quad (42)$$

$$q(t)=\begin{Bmatrix}y(t)\\ \dot{y}(t)\end{Bmatrix} \quad (43)$$

In equation (41), $[I]$ is an identity matrix of the same size as $[M]$ or $[K]$. The vibration modes and periods of vibration were obtained by solving the system of eigenvectors and eigenvalues shown in equation:

$$\left[[K]-w_j^2[M]\right]\{\phi\}_j=0 \quad (44)$$

Where w_j is the j^{th} circular frequency of the building and ϕ_j is the j^{th} mode of vibration. It was not considered the effect of the antenna in the j^{th} periods of vibration, T_j, because the antenna only achieves an increase of 0.55% on the fundamental vibration period. The values of T_j for five modes of vibration of the Latin American Tower are shown in table 4.

Table. 4: Periods of vibration of the Latin American Tower

j^{th} mode of vibration	T_j (s)
1	3.64
2	1.53
3	0.97
4	0.69
5	0.56

The modes of vibration corresponding to Table 4 are shown in figure 7.

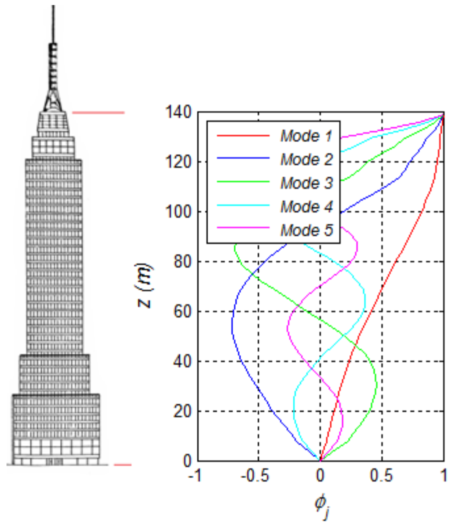

Fig. 7: Modes of vibration of the Latin American Tower

The maximum dynamic responses of each floor of the Latin American Tower are shown in figure 8 to 10.

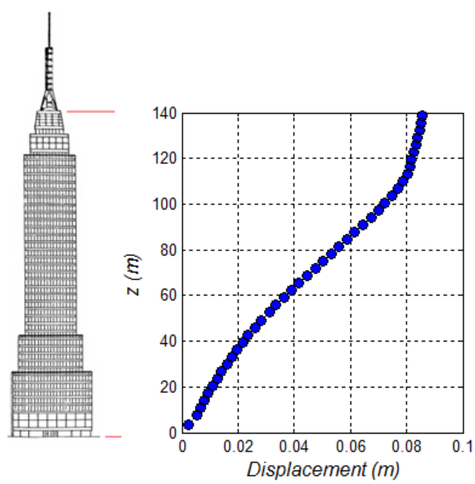

Fig. 8: Maximum displacements of the Latin American Tower

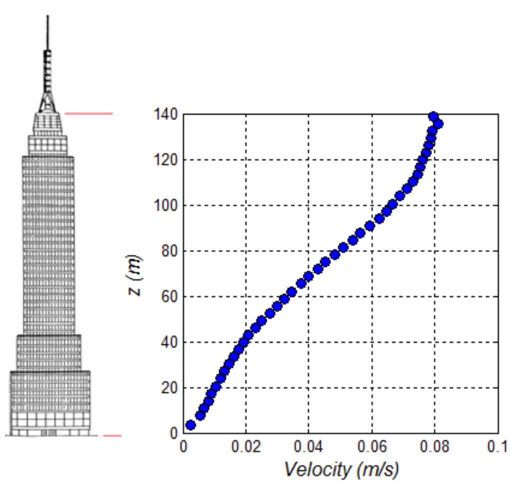

Fig. 9: Maximum velocities of the Latin American Tower

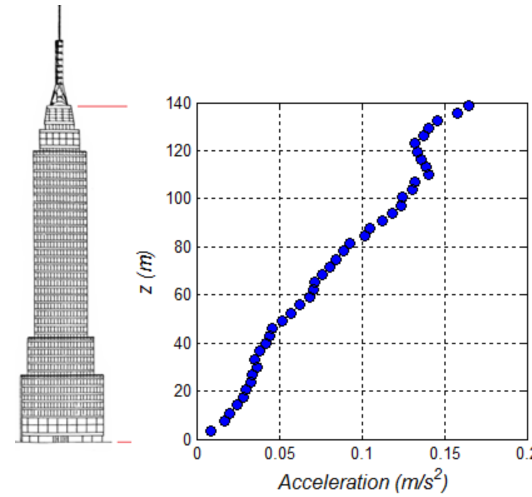

Fig. 10: Maximum accelerations of the Latin American Tower

The maximum shear force at the base, $V_{B,\max}$, and the maximum base overturning moment of the building, $M_{B,\max}$, are shown in table 5 for the dynamic wind forces in the along-wind direction.

Table. 5: Maximum stresses at the base of the Latin American Tower according to the dynamic analysis

$V_{B,\max}$	1,719.14 kN
$M_{B,\max}$	119,875.88 $kN \cdot m$

4. CONCLUSION

The comparison between the static wind analysis made in the 1950s and the along-wind response analysis in time domain is shown in table 6.

Table. 6: Comparison of maximum along-wind response for the Latin American Tower

Maximum response	Static Analysis made in the 1950s	Current dynamic analysis
$y_{\max}$	4.02 cm	8.54 cm
$\dot{y}_{\max}$	-	7.97 cm/s
$\ddot{y}_{\max}$	-	16.47 cm/s^2
$V_{B,\max}$	938.18 kN	1,719.14 kN
$M_{B,\max}$	67,200.62 $kN \cdot m$	119,875.88 $kN \cdot m$

Table 6 shows that the static wind design made in the 1950s for the Latin American Tower underestimated the effects of wind because it did not consider wind turbulence. The dynamic analysis carried out in this paper allows us to consider several modes of vibration, unlike the dynamic amplification coefficient method proposed by the wind design codes. Even so, the consideration of a non-proportional damping matrix and the consideration of the soil-structure interaction will cause changes on the along-wind dynamic response.

The Latin American Tower is able to resist earthquakes of great intensity due to its extraordinary design of foundation, however, the serviceability limit state under wind loads must be reviewed as the years pass due to climate change.

The serviceability limit state under wind loads recommended by The Complementary Technical Norms for wind design of Mexico City [2] is an acceleration of 39.24 cm/s^2. On the other hand, the AIJ [13] proposed a limit value of 29.43 cm/s^2 and the ISO [14] proposed a limit value of 19.62 cm/s^2. It is observed that The Latin American Tower barely satisfy the serviceability limit state proposed by ISO [14].

From the 1950s to the present, great climatic changes have taken place, resulting in increasing winds. For this reason, it is advisable that the design wind codes are constantly being updated to contemplate such increases in the reference wind speed. Many skyscrapers could fail to satisfy the serviceability limit state under wind loads due to impending climate change.

ACKNOWLEDGEMENTS

The accomplishment of this paper was achieved thanks to the valuable contribution of Dr. Hugo Hernández Barrios, who has been able to guide me in an excellent way in my academic training. Thanks also to the Faculty of Engineering of the National Autonomous University of Mexico for being my home during my postgraduate studies.

REFERENCES

[1] Adolfo Zeevaert, Luis Cuevas, "La Torre Latinoamericana", 1ª ed., 84 p., 1983, Mexico.

[2] Department of the City of Mexico, "Normas Técnicas Complementarias para Diseño por Viento", 2004, Mexico.

[3] R.I. Harris and D.M. Deaves, "The structure of strong winds", *Wind Engineering in the Eighties: Proceedings of the CIRIA Conference held on 12-13 November,* paper 4, 1980, England.

[4] British Standard, "BS EN 1991-4:2005, Eurocode 1: Actions on structure, Part 1-4: General actions-Wind Actions", 2005.

[5] G. Solari, "Gust buffeting, I: Peak wind velocity and equivalent pressure", *Journal of Structural Engineering,* vol.119, No.2, 1993.

[6] P. Van Gelder and D. Lungu, "Properties of Gust Spectra with Applications to Wind Loading Codes", *ICOSSAR*'97 *Conference*, 1997, Japan.

[7] G. Solari and G. Piccardo, "Probabilistic 3-d turbulence modeling for gust buffeting of structures", *Probabilistic Engineering Mechanics*, vol.16, pp. 73-86, 2001.

[8] A.G. Davenport, "The prediction of the response of structures to gusty wind", *Safety of Structures under Dynamic Loading,* Holand, Kavlie, Moe and Sigbjornsson (eds.), p. 257-284, 1977, Norway.

[9] Claës Dyrbye and Svend O. Hansen, "Wind Loads on Structures", *(1st ed.), John Wiley & Sons,* 1997.

[10] M. Shinozuka, C.B. Yun, H. Seya, "Stochastic methods in wind engineering", *J.Wind Eng. Ind. Aerdyn,* vol. 36, pp. 829-843, 1990.

[11] H.G. Castro, M.E. Bortoli, R.R. Paz and J.O. Marighetti, "Una metodología de cálculo para la determinación de la respuesta dinámica longitudinal de estructuras altas bajo la acción del viento", *Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería*, vol. 31(4), pp. 235-245, 2015.

[12] B.J. Vickery, "Load fluctuations in turbulent flow", *Proc. of ASEC, EM1*, 1968.

[13] AIJ, "Architectural Institute of Japan Recommendations for Loads on Buildings", *Chapter 6, Wind Loads*, 1994, Japan.

[14] ISO/TC98/SC3/WG2, "Wind Loading (Static and Dynamic)", *International Standard Organization*, 1991, Switzerland.